CHAPTER 9 | ROTATIONAL DYNAMICS

PROBLEMS

1. **REASONING AND SOLUTION** Solving Equation 9.1 for the lever arm \( \ell \), we obtain

\[
\ell = \frac{\tau}{F} = \frac{3.0 \text{ N} \cdot \text{m}}{12 \text{ N}} = 0.25 \text{ m}
\]

2. **REASONING** The torque is given by Equation 9.1, \( \tau = F \ell \), where \( F \) is the magnitude of the applied force and \( \ell \) is the lever arm. From the figure in the text, the lever arm is given by \( \ell = (0.28 \text{ m}) \sin 50.0^\circ \). Since both \( \tau \) and \( \ell \) are known, Equation 9.1 can be solved for \( F \).

**SOLUTION** Solving Equation 9.1 for \( F \), we have

\[
F = \frac{\tau}{\ell} = \frac{45 \text{ N} \cdot \text{m}}{(0.28 \text{ m}) \sin 50.0^\circ} = 2.1 \times 10^2 \text{ N}
\]

3. **REASONING AND SOLUTION** The torque about the center of the cable car is \( \tau = FL \), where \( L \) is the lever arm.

\[
\tau = FL = 2(185 \text{ N})(4.60 \text{ m}) = 1.70 \times 10^3 \text{ N} \cdot \text{m}
\]

4. **REASONING** To calculate the torques, we need to determine the lever arms for each of the forces. These lever arms are shown in the following drawings:

**SOLUTION**

a. Using Equation 9.1, we find that the magnitude of the torque due to the weight \( W \) is
b. Using Equation 9.1, we find that the magnitude of the torque due to the thrust \( T \) is

\[
\tau_T = T\ell_T = 2\,300 \text{ N} \cdot 0.5 \text{ m} \cdot \cos 32^\circ = 132,000 \text{ N} \cdot \text{m}
\]

5. **REASONING** The torque on either wheel is given by Equation 9.1, \( \tau = F\ell \), where \( F \) is the magnitude of the force and \( \ell \) is the lever arm. Regardless of how the force is applied, the lever arm will be proportional to the radius of the wheel.

**SOLUTION** The ratio of the torque produced by the force in the truck to the torque produced in the car is

\[
\frac{\tau_{\text{truck}}}{\tau_{\text{car}}} = \frac{F\ell_{\text{truck}}}{F\ell_{\text{car}}} = \frac{Fr_{\text{truck}}}{Fr_{\text{car}}} = \frac{r_{\text{truck}}}{r_{\text{car}}} = \frac{0.25 \text{ m}}{0.19 \text{ m}} = 1.3
\]

6. **REASONING** To calculate the torque, we will need the lever arm of the 110 N force. The drawing identifies the lever arm \( \ell \).

**SOLUTION** According to Equation 9.1, the magnitude of the torque is

\[
\tau = W\ell = 110 \text{ N} \cdot 3.0 \text{ m} \cos 15^\circ = 320 \text{ N} \cdot \text{m}
\]

7. **REASONING AND SOLUTION** The torque due to \( F_1 \) is

\[
\tau_1 = -F_1\ell_1 = -(20.0 \text{ N})(0.500 \text{ m}) = -10.0 \text{ N} \cdot \text{m} \text{ (CW)}
\]

The torque due to \( F_2 \) is

\[
\tau_2 = (F_2 \cos 30.0^\circ)\ell_2 = (35.0 \text{ N})(1.10 \text{ m}) \cos 30.0^\circ = 33.3 \text{ N} \cdot \text{m} \text{ (CCW)}
\]

The net torque is therefore,
\[ \Sigma \tau = \tau_1 + \tau_2 = -10.0 \text{ N} \cdot \text{m} + 33.3 \text{ N} \cdot \text{m} = 23.3 \text{ N} \cdot \text{m}, \text{ counterclockwise} \]

8. **REASONING AND SOLUTION**  The torque produced by a force of magnitude \( F \) is given by Equation 9.1, \( \tau = F \ell \), where \( \ell \) is the lever arm. In each case, the torque produced by the couple is equal to the sum of the individual torques produced by each member of the couple.

a. When the axis passes through point \( A \), the torque due to the force at \( A \) is zero. The lever arm for the force at \( C \) is \( L \). Therefore, taking counterclockwise as the positive direction, we have
\[
\tau = \tau_A + \tau_C = 0 + \tau_C = FL
\]

b. Each force produces a counterclockwise rotation. The magnitude of each force is \( F \) and each force has a lever arm of \( L/2 \). Taking counterclockwise as the positive direction, we have
\[
\tau = \tau_A + \tau_C = F \left( \frac{L}{2} \right) + F \left( \frac{L}{2} \right) = FL
\]

c. When the axis passes through point \( C \), the torque due to the force at \( C \) is zero. The lever arm for the force at \( A \) is \( L \). Therefore, taking counterclockwise as the positive direction, we have
\[
\tau = \tau_A + \tau_C = \tau_A + 0 = FL
\]

Note that the value of the torque produced by the couple is the same in all three cases; in other words, when the couple acts on the tire wrench, the couple produces a torque that does not depend on the location of the axis.

9. **REASONING**  Since the meter stick does not move, it is in equilibrium. The forces and the torques, therefore, each add to zero. We can determine the location of the 6.00 N force, by using the condition that the sum of the torques must vectorially add to zero.

**SOLUTION**  If we take counterclockwise torques as positive, then the torque of the first force about the pin is
\[
\tau_1 = F_1 \ell_1 = (2.00 \text{ N})(1.00 \text{ m}) = 2.00 \text{ N} \cdot \text{m}
\]
The torque due to the second force is

$$\tau_2 = -F_2 \sin 30.0^\circ \ell_2 = -(6.00 \text{ N})(\sin 30.0^\circ) \ell_2 = -(3.00 \text{ N}) \ell_2$$

The torque $\tau_2$ is negative because the force $F_2$ tends to produce a clockwise rotation about the pinned end. Since the net torque is zero, we have

$$2.00 \text{ N} \cdot \text{m} + [-(3.00 \text{ N}) \ell_2] = 0$$

Thus,

$$\ell_2 = \frac{2.00 \text{ N} \cdot \text{m}}{3.00 \text{ N}} = 0.667 \text{ m}$$

10. **REASONING AND SOLUTION** The net torque about the axis in text drawing (a) is

$$\Sigma \tau = \tau_1 + \tau_2 = F_1 b - F_2 a = 0.$$  

Considering that $F_2 = 3F_1$, we have $b - 3a = 0$. The net torque in drawing (b) is then

$$\Sigma \tau = F_1 (1.00 \text{ m} - a) - F_2 b = 0 \quad \text{or} \quad 1.00 \text{ m} - a - 3b = 0.$$  

Solving the first equation for $b$, substituting into the second equation and rearranging, gives

$$a = 0.100 \text{ m}$$  

and

$$b = 0.300 \text{ m}$$

11. **REASONING AND SOLUTION**

a. Taking torques about an axis through the rear axle of the car gives

$$2F_1(2.54 \text{ m}) - (1160 \text{ kg})(9.80 \text{ m/s}^2)(1.52 \text{ m}) = 0 \quad \text{or} \quad F_1 = 3.40 \times 10^3 \text{ N}$$

b. Taking torques about an axis through the front axle gives

$$(1160 \text{ kg})(9.80 \text{ m/s}^2)(1.02 \text{ m}) - 2F_1(2.54 \text{ m}) = 0 \quad \text{or} \quad F_1 = 2.28 \times 10^3 \text{ N}$$

12. **REASONING** The drawing shows the forces acting on the person. It also shows the lever arms for a rotational axis
perpendicular to the plane of the paper at the place where the person’s toes touch the floor. Since the person is in equilibrium, the sum of the forces must be zero. Likewise, we know that the sum of the torques must be zero.

**SOLUTION** Taking upward to be the positive direction, we have

\[ F_{\text{FEET}} + F_{\text{HANDS}} - W = 0 \]

Remembering that counterclockwise torques are positive and using the axis and the lever arms shown in the drawing, we find

\[ W\ell_{W} - F_{\text{HANDS}}\ell_{\text{HANDS}} = 0 \]

\[ F_{\text{HANDS}} = \frac{W\ell_{W}}{\ell_{\text{HANDS}}} = \frac{384 \text{ N} \cdot 840 \text{ m}}{1.250 \text{ m}} = 392 \text{ N} \]

Substituting this value into the balance-of-forces equation, we find

\[ F_{\text{FEET}} = W - F_{\text{HANDS}} = 584 \text{ N} - 392 \text{ N} = 192 \text{ N} \]

The force on each hand is half the value calculated above, or \[ 196 \text{ N} \]. Likewise, the force on each foot is half the value calculated above, or \[ 96 \text{ N} \].

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13. **REASONING** The minimum value for the coefficient of static friction between the ladder and the ground, so that the ladder does not slip, is given by Equation 4.7:

\[ f_s^{\text{MAX}} = \mu_s F_N \]

**SOLUTION** From Example 4, the magnitude of the force of static friction is \[ G_x = 727 \text{ N} \]. The magnitude of the normal force applied to the ladder by the ground is \[ G_y = 1230 \text{ N} \]. The minimum value for the coefficient of static friction between the ladder and the ground is

\[ \mu_s = \frac{f_s^{\text{MAX}}}{F_N} = \frac{G_x}{G_y} = \frac{727 \text{ N}}{1230 \text{ N}} = 0.591 \]
14. **REASONING** When the board just begins to tip, three forces act on the board. They are the weight $W$ of the board, the weight $W_p$ of the person, and the force $F$ exerted by the right support.

Since the board will rotate around the right support, the lever arm for this force is zero, and the torque exerted by the right support is zero. The lever arm for the weight of the board is equal to one-half the length of the board minus the overhang length: $2.5 \text{ m} - 1.1 \text{ m} = 1.4 \text{ m}$.

The lever arm for the weight of the person is $x$. Therefore, taking counterclockwise torques as positive, we have

$$-W_p x + W (1.4 \text{ m}) = 0$$

This expression can be solved for $x$.

**SOLUTION** Solving the expression above for $x$, we obtain

$$x = \frac{W (1.4 \text{ m})}{W_p} = \frac{(225 \text{ N})(1.4 \text{ m})}{450 \text{ N}} = 0.70 \text{ m}$$
15. **REASONING** The figure at the right shows the door and the forces that act upon it. Since the door is uniform, the center of gravity, and, thus, the location of the weight \( W \), is at the geometric center of the door.

Let \( U \) represent the force applied to the door by the upper hinge, and \( L \) the force applied to the door by the lower hinge. Taking forces that point to the right and forces that point up as positive, we have

\[
\sum F_x = L_x - U_x = 0 \quad \text{or} \quad L_x = U_x \tag{1}
\]
\[
\sum F_y = L_y - W = 0 \quad \text{or} \quad L_y = W = 140 \text{ N} \tag{2}
\]

Taking torques about an axis perpendicular to the plane of the door and through the lower hinge, with counterclockwise torques being positive, gives

\[
\sum \tau = -W\left(\frac{d}{2}\right) + U_xh = 0 \tag{3}
\]

Equations (1), (2), and (3) can be used to find the force applied to the door by the hinges; Newton's third law can then be used to find the force applied by the door to the hinges.

**SOLUTION**

a. Solving Equation (3) for \( U_x \) gives

\[
U_x = \frac{Wd}{2h} = \frac{(140 \text{ N})(0.81 \text{ m})}{2(2.1 \text{ m})} = 27 \text{ N}
\]

From Equation (1), \( L_x = 27 \text{ N} \).

Therefore, the upper hinge exerts on the door a horizontal force of **27 N to the left**.

b. From Equation (1), \( L_x = 27 \text{ N} \), we conclude that the bottom hinge exerts on the door a horizontal force of **27 N to the right**.

c. From Newton's third law, the door applies a force of **27 N** on the upper hinge. This force **points horizontally to the right**, according to Newton's third law.
d. Let $D$ represent the force applied by the door to the lower hinge. Then, from Newton’s third law, the force exerted on the lower hinge by the door has components $D_x = -27 \text{ N}$ and $D_y = -140 \text{ N}$.

From the Pythagorean theorem,

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-27 \text{ N})^2 + (-140 \text{ N})^2} = 143 \text{ N}$$

The angle $\theta$ is given by

$$\theta = \tan^{-1} \left( \frac{140 \text{ N}}{-27 \text{ N}} \right) = 79^\circ$$

The force is directed $79^\circ$ below the horizontal.

16. **REASONING AND SOLUTION**  

The net torque about an axis through the contact point between the tray and the thumb is

$$\Sigma \tau = F(0.0400 \text{ m}) - (0.250 \text{ kg})(9.80 \text{ m/s}^2)(0.320 \text{ m}) - (1.00 \text{ kg})(9.80 \text{ m/s}^2)(0.180 \text{ m})$$

$$- (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0.140 \text{ m}) = 0$$

$$F = 70.6 \text{ N, up}$$

Similarly, the net torque about an axis through the point of contact between the tray and the finger is

$$\Sigma \tau = T(0.0400 \text{ m}) - (0.250 \text{ kg})(9.80 \text{ m/s}^2)(0.280 \text{ m}) - (1.00 \text{ kg})(9.80 \text{ m/s}^2)(0.140 \text{ m})$$

$$- (0.200 \text{ kg})(9.80 \text{ m/s}^2)(0.100 \text{ m}) = 0$$

$$T = 56.4 \text{ N, down}$$

17. **REASONING AND SOLUTION**  

When the arm is in equilibrium, the sum of the torques is zero. Taking torques about the elbow joint with counterclockwise torques taken as positive, we have

$$- F \ell_F + M \ell_M = 0$$

Solving for $M$,

$$M = F \left( \frac{\ell_F}{\ell_M} \right) = (170 \text{ N}) \left( \frac{0.30 \text{ m}}{0.051 \text{ m}} \right) = 1.0 \times 10^3 \text{ N}$$

The direction of the force is to the left.
18. **REASONING** Although this arrangement of body parts is vertical, we can apply Equation 9.3 to locate the overall center of gravity by simply replacing the horizontal position \(x\) by the vertical position \(y\), as measured relative to the floor.

**SOLUTION** Using Equation 9.3, we have

\[
y_{cg} = \frac{W_1y_1 + W_2y_2 + W_3y_3}{W_1 + W_2 + W_3}
\]

\[
y = \frac{0.38 \text{ N} \cdot 0.28 \text{ m} + 0.44 \text{ N} \cdot 0.760 \text{ m} + 0.767 \text{ N} \cdot 0.250 \text{ m}}{438 \text{ N} + 144 \text{ N} + 87 \text{ N}} = 1.03 \text{ m}
\]

19. **REASONING**

The jet is in equilibrium, so the sum of the external forces is zero, and the sum of the external torques is zero. We can use these two conditions to evaluate the forces exerted on the wheels.

**SOLUTION**

a. Let \(F_f\) be the magnitude of the normal force that the ground exerts on the front wheel. Since the net torque acting on the plane is zero, we have (using an axis through the points of contact between the rear wheels and the ground)

\[
\Sigma \tau = -W \ell_w + F_f \ell_f = 0
\]

where \(W\) is the weight of the plane, and \(\ell_w\) and \(\ell_f\) are the lever arms for the forces \(W\) and \(F_f\), respectively. Thus,

\[
\Sigma \tau = -(1.00 \times 10^6 \text{ N})(15.0 \text{ m} - 12.6 \text{ m}) + F_f (15.0 \text{ m}) = 0
\]

Solving for \(F_f\) gives \(F_f = \frac{1.60 \times 10^5 \text{ N}}{}\).

b. Setting the sum of the vertical forces equal to zero yields

\[
\Sigma F_y = F_f + 2F_f - W = 0
\]

where the factor of 2 arises because there are two rear wheels. Substituting in the data,

\[
\Sigma F_y = 1.60 \times 10^5 \text{ N} + 2F_f - 1.00 \times 10^6 \text{ N} = 0
\]
20. **REASONING AND SOLUTION**
   
a. Taking torques about an axis through the bolt we have

\[-(4.5 \times 10^3 \text{ N})(2.50 \text{ m}) - (12.0 \times 10^3 \text{ N})(3.50 \text{ m}) + T(5.00 \text{ m}) \sin 25.0^\circ = 0\]

Solving for \(T\) yields

\[T = 2.52 \times 10^4 \text{ N}\]

b. Applying Newton's second law in the horizontal direction gives \(-T \cos 25.0^\circ + F_h = 0\), so

\[F_h = 2.28 \times 10^4 \text{ N}\]

In the vertical direction \(F_v - W_L - W_B + T \sin 25.0^\circ = 0\), and

\[F_v = 5.85 \times 10^3 \text{ N}\]

Now the force exerted on the beam by the bolt is

\[F = \sqrt{F_h^2 + F_v^2} = 2.35 \times 10^4 \text{ N}\]

21. **REASONING AND SOLUTION** The figure below shows the massless board and the forces that act on the board due to the person and the scales.

![Diagram of a massless board with forces](image)

a. Applying Newton's second law to the vertical direction gives

\[315 \text{ N} + 425 \text{ N} - W = 0 \quad \text{or} \quad W = 7.40 \times 10^2 \text{ N, downward}\]
b. Let \( x \) be the position of the center of gravity relative to the scale at the person’s head. Taking torques about an axis through the contact point between the scale at the person’s head and the board gives

\[
(315 \text{ N})(2.00 \text{ m}) - (7.40 \times 10^2 \text{ N})x = 0 \quad \text{or} \quad x = 0.851 \text{ m}
\]

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22. **REASONING AND SOLUTION**  The net torque about an axis through the knee joint is

\[
\Sigma \tau = M(\sin 25.0^\circ)(0.100 \text{ m}) - (44.5 \text{ N})(\cos 30.0^\circ)(0.250 \text{ m}) = 0
\]

\[
M = 228 \text{ N}
\]

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23. **SSM  REASONING**  Since the man holds the ball motionless, the ball and the arm are in equilibrium. Therefore, the net force, as well as the net torque about any axis, must be zero.

**SOLUTION**

a. Using Equation 9.1, the net torque about an axis through the elbow joint is

\[
\Sigma \tau = M(0.0510 \text{ m}) - (22.0 \text{ N})(0.140 \text{ m}) - (178 \text{ N})(0.330 \text{ m}) = 0
\]

Solving this expression for \( M \) gives \( M = 1.21 \times 10^3 \text{ N} \).

b. The net torque about an axis through the center of gravity is

\[
\Sigma \tau = -(1210 \text{ N})(0.0890 \text{ m}) + F(0.140 \text{ m}) - (178 \text{ N})(0.190 \text{ m}) = 0
\]

Solving this expression for \( F \) gives \( F = 1.01 \times 10^3 \text{ N} \). Since the forces must add to give a net force of zero, we know that the direction of \( F \) is **downward**.

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24. **REASONING**  If we assume that the system is in equilibrium, we know that the vector sum of all the forces, as well as the vector sum of all the torques, that act on the system must be zero.

The figure below shows a free body diagram for the boom. Since the boom is assumed to be uniform, its weight \( W_B \) is located at its center of gravity, which coincides with its geometrical center. There is a tension \( T \) in the cable that acts at an angle \( \theta \) to the horizontal, as shown. At the hinge pin \( P \), there are two forces acting. The vertical force \( V \) that acts on the end of the boom
prevents the boom from falling down. The horizontal force $H$ that also acts at the hinge pin prevents the boom from sliding to the left. The weight $W_L$ of the wrecking ball (the "load") acts at the end of the boom.

![Diagram of boom with forces and angles]

By applying the equilibrium conditions to the boom, we can determine the desired forces.

**SOLUTION** The directions upward and to the right will be taken as the positive directions. In the $x$ direction we have

$$\sum F_x = H - T \cos \Theta = 0 \quad (1)$$

while in the $y$ direction we have

$$\sum F_y = V - T \sin \Theta - W_L - W_B = 0 \quad (2)$$

Equations (1) and (2) give us two equations in three unknown. We must, therefore, find a third equation that can be used to determine one of the unknowns. We can get the third equation from the torque equation.

In order to write the torque equation, we must first pick an axis of rotation and determine the lever arms for the forces involved. Since both $V$ and $H$ are unknown, we can eliminate them from the torque equation by picking the rotation axis through the point $P$ (then both $V$ and $H$ have zero lever arms). If we let the boom have a length $L$, then the lever arm for $W_L$ is $L \cos \Phi$, while the lever arm for $W_B$ is $(L/2) \cos \Phi$. From the figure, we see that the lever arm for $T$ is $L \sin (\Phi - \Theta)$. If we take counterclockwise torques as positive, then the torque equation is

$$\sum \tau = -W_B \left( \frac{L \cos \Phi}{2} \right) - W_L \cos \Phi + TL \sin (\Phi - \Theta) = 0$$

Solving for $T$, we have

$$T = \frac{\frac{1}{2} W_B + W_L}{\sin (\Phi - \Theta)} \cos \Phi \quad (3)$$
a. From Equation (3) the tension in the support cable is

\[ T = \frac{1}{2} \frac{(3600 \text{ N}) + 4800 \text{ N}}{\sin(48^\circ - 32^\circ)} \cos 48^\circ = 1.6 \times 10^4 \text{ N} \]

b. The force exerted on the lower end of the hinge at the point \( P \) is the vector sum of the forces \( H \) and \( V \). According to Equation (1),

\[ H = T \cos \Theta = \left(1.6 \times 10^4 \text{ N}\right) \cos 32^\circ = 1.4 \times 10^4 \text{ N} \]

and, from Equation (2)

\[ V = W_L + W_B + T \sin \Theta = 4800 \text{ N} + 3600 \text{ N} + \left(1.6 \times 10^4 \text{ N}\right) \sin 32^\circ = 1.7 \times 10^4 \text{ N} \]

Since the forces \( H \) and \( V \) are at right angles to each other, the magnitude of their vector sum can be found from the Pythagorean theorem:

\[ F_p = \sqrt{H^2 + V^2} = \sqrt{(1.4 \times 10^4 \text{ N})^2 + (1.7 \times 10^4 \text{ N})^2} = 2.2 \times 10^4 \text{ N} \]

25. **REASONING AND SOLUTION**  Taking torques on the right leg about the apex of the “A” gives

\[ \sum \tau = -(120 \text{ N}) L \cos 60.0^\circ - FL \cos 30.0^\circ + T(2L) \cos 60.0^\circ = 0 \]

where \( F \) is the horizontal component of the force exerted by the crossbar and \( T \) is the tension in the right string. Due to symmetry, the tension in the left string is also \( T \). Newton’s second law applied to the vertical gives

\[ 2T - 2W = 0 \quad \text{so} \quad T = 120 \text{ N} \]

Substituting \( T = 120 \text{ N} \) into the first equation yields \( F = 69 \text{ N} \).
26. **REASONING** The drawing shows the forces acting on the board, which has a length \( L \). Wall 2 exerts a normal force \( P_2 \) on the lower end of the board. The maximum force of static friction that wall 2 can apply to the lower end of the board is \( \mu_s P_2 \) and is directed upward in the drawing. The weight \( W \) acts downward at the board’s center. Wall 1 applies a normal force \( P_1 \) to the upper end of the board. We take upward and to the right as our positive directions.

**SOLUTION** Then, since the horizontal forces balance to zero, we have

\[
P_1 - P_2 = 0
\]

(1)

The vertical forces also balance to zero:

\[
\mu_s P_2 - W = 0
\]

(2)

Using an axis through the lower end of the board, we now balance the torques to zero:

\[
W \left( \frac{L}{2} \right) (\cos \theta) - P_1 L (\sin \theta) = 0
\]

(3)

Rearranging Equation (3) gives

\[
tan \theta = \frac{W}{2P_1}
\]

(4)

But \( W = \mu_s P_2 \) according to Equation (2), and \( P_2 = P_1 \) according to Equation (1). Therefore, \( W = \mu_s P_1 \), which can be substituted in Equation (4) to show that

\[
tan \theta = \frac{\mu_s P_1}{2P_1} = \frac{\mu_s}{2} = \frac{0.98}{2}
\]

or

\[
\theta = tan^{-1}(0.49) = 26^\circ
\]
From the drawing at the right,
\[ \cos \theta = \frac{d}{L} \]

Therefore, the longest board that can be propped between the two walls is
\[ L = \frac{d}{\cos \theta} = \frac{1.5 \text{ m}}{\cos 26^\circ} = 1.7 \text{ m} \]

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27. **REASONING AND SOLUTION** The weight \( W \) of the left side of the ladder, the normal force \( F_N \) of the floor on the left leg of the ladder, the tension \( T \) in the crossbar, and the reaction force \( R \) due to the right-hand side of the ladder, are shown in the figure below. In the vertical direction \(-W + F_N = 0\), so that
\[ F_N = W = mg = (10.0 \text{ kg})(9.80 \text{ m/s}^2) = 98.0 \text{ N} \]

In the horizontal direction it is clear that \( R = T \).
The net torque about the base of the ladder is
\[ \Sigma \tau = -T [(1.00 \text{ m}) \sin 75.0^\circ] - W [(2.00 \text{ m}) \cos 75.0^\circ] + R [(4.00 \text{ m}) \sin 75.0^\circ] = 0 \]

Substituting for \( W \) and using \( R = T \), we obtain
\[ T = \frac{(98.0 \text{ N})(2.00 \text{ m}) \cos 75.0^\circ}{(3.00 \text{ m}) \sin 75.0^\circ} = 17.5 \text{ N} \]

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28. **REASONING** The moment of inertia of the stool is the sum of the individual moments of inertia of its parts. According to Table 9.1, a circular disk of radius \( R \) has a moment of inertia of \( I_{\text{disk}} = \frac{1}{2} M_{\text{disk}} R^2 \) with respect to an axis perpendicular to the disk center. Each thin rod is attached perpendicular to the disk at its outer edge. Therefore, each particle in a rod is located at a perpendicular distance from the axis that is equal to the radius of the disk. This means that each of the rods has a moment of inertia of \( I_{\text{rod}} = M_{\text{rod}} R^2 \).

**SOLUTION** Remembering that the stool has three legs, we find that the its moment of inertia is
29. **REASONING AND SOLUTION** For a rigid body rotating about a fixed axis, Newton's second law for rotational motion is given by Equation 9.7, \( \sum \tau = I \alpha \), where \( I \) is the moment of inertia of the body and \( \alpha \) is the angular acceleration expressed in rad/s\(^2\). Equation 9.7 gives

\[
I = \frac{\sum \tau}{\alpha} = \frac{10.0 \text{ N} \cdot \text{m}}{8.00 \text{ rad/s}^2} = \frac{1.25 \text{ kg} \cdot \text{m}^2}{2}
\]

---

30. **REASONING AND SOLUTION** We know

\[
\sum \tau = I \alpha = (0.16 \text{ kg} \cdot \text{m}^2)(7.0 \text{ rad/s}^2) = 1.1 \text{ kg} \cdot \text{m}^2/\text{s}^2 = 1.1 \text{ N} \cdot \text{m}
\]

---

31. **REASONING AND SOLUTION**
   a. The net torque on the disk about the axle is

\[
\sum \tau = F_1 R - F_2 R = (0.314 \text{ m})(90.0 \text{ N} - 125 \text{ N}) = -11 \text{ N} \cdot \text{m}
\]

   b. The angular acceleration is given by \( \alpha = \sum \tau/I \). From Table 9.1, the moment of inertia of the disk is

\[
I = (1/2) MR^2 = (1/2)(24.3 \text{ kg})(0.314 \text{ m})^2 = 1.20 \text{ kg} \cdot \text{m}^2
\]

\[
\alpha = (-11 \text{ N} \cdot \text{m})/(1.20 \text{ kg} \cdot \text{m}^2) = -9.2 \text{ rad} / \text{s}^2
\]

---

32. **REASONING** The rotational analog of Newton’s second law is given by Equation 9.7, \( \sum \tau = I \alpha \). Since the person pushes on the outer edge of one pane of the door with a force \( F \) that is directed perpendicular to the pane, the torque exerted on the door has a magnitude of \( FL \), where the lever arm \( L \) is equal to the width of one pane of the door. Once the moment of inertia is known, Equation 9.7 can be solved for the angular acceleration \( \alpha \).
The moment of inertia of the door relative to the rotation axis is $I = 4I_p$, where $I_p$ is the moment of inertia for one pane. According to Table 9.1, we find $I_p = \frac{1}{3} ML^2$, so that the rotational inertia of the door is $I = \frac{4}{3} ML^2$.

**SOLUTION** Solving Equation 9.7 for $\alpha$, and using the expression for $I$ determined above, we have

$$\alpha = \frac{FL}{\frac{4}{3} ML^2} = \frac{F}{\frac{4}{3} ML} = \frac{68 \text{ N}}{\frac{4}{3} (85 \text{ kg})(1.2 \text{ m})} = 0.50 \text{ rad/s}^2$$

33. **REASONING** The figure below shows eight particles, each one located at a different corner of an imaginary cube. As shown, if we consider an axis that lies along one edge of the cube, two of the particles lie on the axis, and for these particles $r = 0$. The next four particles closest to the axis have $r = \ell$, where $\ell$ is the length of one edge of the cube. The remaining two particles have $r = d$, where $d$ is the length of the diagonal along any one of the faces. From the Pythagorean theorem, $d = \sqrt{\ell^2 + \ell^2} = \ell \sqrt{2}$.

![Diagram of an imaginary cube with particles at corners](image)

According to Equation 9.6, the moment of inertia of a system of particles is given by $I = \sum mr^2$.

**SOLUTION** Direct application of Equation 9.6 gives

$$I = \sum mr^2 = 4(m \ell^2) + 2(md^2) = 4(m \ell^2) + 2(2m \ell^2) = 8m \ell^2$$

or

$$I = 8(0.12 \text{ kg})(0.25 \text{ m})^2 = 0.060 \text{ kg} \cdot \text{m}^2$$

34. **REASONING AND SOLUTION**
a. We know that \( \alpha = (\omega - \omega_0)/t \). But \( \omega = (1/2)\omega_0 \), so

\[
\alpha = -(1/2)\omega_0/t = -(1/2)(88.0 \text{ rad/s})/(5.00 \text{ s}) = -8.80 \text{ rad/s}^2
\]

The magnitude of \( \alpha \) is \( 8.80 \text{ rad/s}^2 \).

b. \( \Sigma \tau = I \alpha \) and \( FR = I \alpha \). The magnitude of the frictional force is

\[
F = I \alpha/R = (0.850 \text{ kg m}^2)(8.80 \text{ rad/s}^2)/(0.0750 \text{ m}) = 99.7 \text{ N}
\]

35. **REASONING AND SOLUTION** We know \( \Sigma \tau = I \alpha \), where

\[
I = (1/2) m r^2 = (1/2)(0.400 \text{ kg})(0.130 \text{ m})^2 = 3.38 \times 10^{-3} \text{ kg m}^2
\]

Also, \( \omega = \omega_0 + \alpha t \), so that \( \alpha = (85.0 \text{ rad/s} - 262 \text{ rad/s})/(18.0 \text{ s}) = -9.83 \text{ rad/s}^2 \). Thus, the net torque is

\[
\Sigma \tau = I \alpha = (3.38 \times 10^{-3} \text{ kg m}^2)(-9.83 \text{ rad/s}^2) = -3.32 \times 10^{-2} \text{ N m}
\]

36. **REASONING** The force applied to the baton creates a net torque that gives the rod an angular acceleration, according to Newton’s second law as expressed for rotational motion \( \Sigma \tau = I \alpha \). From the data given, we will calculate the moment of inertia \( I \) and the angular acceleration \( \alpha \). Newton’s second law, then, will allow us to obtain the net torque \( \Sigma \tau \). From the net torque, we will be able to determine the force, since the magnitude of the torque in this case is just the magnitude of the force times the lever arm of 2.0 cm.

**SOLUTION** The momentum of inertia of the baton is the sum of the individual moments of inertia of its parts. For a thin rod of length \( L \) rotating about an axis perpendicular to the rod at its center, the moment of inertia is \( I_{\text{rod}} = \frac{1}{12} M_{\text{rod}} L^2 \). For each ball (assumed to be very small compared to the length of the rod), the moment of inertia is \( I_{\text{ball}} = M_{\text{ball}} (L/2)^2 \). The total moment of inertia, then, is

\[
I_{\text{baton}} = I_{\text{rod}} + 2 I_{\text{ball}} = \frac{1}{12} M_{\text{rod}} L^2 + 2 M_{\text{ball}} \frac{L}{2} = \frac{1}{12} M_{\text{rod}} L^2 + \frac{1}{2} M_{\text{ball}} L^2
\]

Using Equation 8.4 from the equations of kinematics for rotational motion, we can determine the angular acceleration as
\[ \alpha = \frac{\omega - \omega_0}{t} \]

where \( \omega \) and \( \omega_0 \) are, respectively, the final and initial angular velocities. Since only one force of magnitude \( F \) creates the torque and since Equation 9.1 gives the magnitude of the torque as the magnitude of the force times the lever arm \( \ell \), Newton’s second law becomes \( \Sigma \tau = F \ell = I \alpha \).

Substituting our expressions for the moment of inertia and angular acceleration of the baton into the second law gives

\[ F \ell = I_{\text{baton}} \alpha = \frac{G}{T} M_{\text{rod}} L^2 + \frac{1}{2} M_{\text{ball}} L^2 \]

In Newton’s second law the angular acceleration \( \alpha \) must be expressed in \( \text{rad/s}^2 \). Therefore, the values for the angular speeds \( \omega \) and \( \omega_0 \) must be expressed in \( \text{rad/s} \). Recognizing that 2.0 rev/s is 4.0 \( \pi \) rad/s and solving for \( F \), we find that

\[ F = \frac{G}{T} M_{\text{rod}} L^2 + \frac{1}{2} M_{\text{ball}} L^2 \]

\[ = \frac{1}{12} \cdot 0.13 \text{ kg} \cdot 0.81 \text{ m}^2 + \frac{1}{2} \cdot 0.13 \text{ kg} \cdot 0.81 \text{ m}^2 \cdot \pi \text{ rad/s} - 0 \text{ rad/s} \]

\[ = 78 \text{ N} \]

---

37. **REASONING AND SOLUTION**

a. From Table 9.1 in the text, the moment of inertia of the rod relative to an axis that is perpendicular to the rod at one end is given by

\[ I_{\text{rod}} = \frac{1}{3} ML^2 = \frac{1}{3} (2.00 \text{ kg})(2.00 \text{ m})^2 = 2.67 \text{ kg} \cdot \text{m}^2 \]

b. The moment of inertia of a point particle of mass \( m \) relative to a rotation axis located a perpendicular distance \( r \) from the particle is \( I = mr^2 \). Suppose that all the mass of the rod were located at a single point located a perpendicular distance \( R \) from the axis in part (a). If this point particle has the same moment of inertia as the rod in part (a), then the distance \( R \), the radius of gyration, is given by

\[ R = \sqrt{\frac{I}{m}} = \sqrt{\frac{2.67 \text{ kg} \cdot \text{m}^2}{2.00 \text{ kg}}} = 1.16 \text{ m} \]
38. **REASONING AND SOLUTION** The final angular speed of the arm is \( \omega = \frac{v_T}{r} \), where \( r = 0.28 \) m. The angular acceleration needed to produce this angular speed is \( \alpha = \frac{\omega - \omega_0}{t} \). The net torque required is \( \Sigma \tau = I \alpha \). This torque is due solely to the force \( M \), so that \( \Sigma \tau = ML \). Thus,

\[
M = \frac{\Sigma \tau}{L} = \frac{I}{t} \left( \frac{\omega - \omega_0}{t} \right)
\]

Setting \( \omega_0 = 0 \) and \( \omega = \frac{v_T}{r} \), the force becomes

\[
M = \frac{I}{L} \left( \frac{v_T}{r} \right) = \frac{I}{L} \left( \frac{0.065 \text{ kg} \cdot \text{m}^2}{0.28 \text{ m}} \right) \left( \frac{0.10 \text{ m/s}}{0.28 \text{ m}} \right) = 460 \text{ N}
\]

39. **REASONING** The angular acceleration of the bicycle wheel can be calculated from Equation 8.4. Once the angular acceleration is known, Equation 9.7 can be used to find the net torque caused by the brake pads. The normal force can be calculated from the torque using Equation 9.1.

**SOLUTION** The angular acceleration of the wheel is, according to Equation 8.4,

\[
\alpha = \frac{\omega - \omega_0}{t} = \frac{3.7 \text{ rad/s} - 13.1 \text{ rad/s}}{3.0 \text{ s}} = -3.1 \text{ rad/s}^2
\]

If we assume that all the mass of the wheel is concentrated in the rim, we may treat the wheel as a hollow cylinder. From Table 9.1, we know that the moment of inertia of a hollow cylinder of mass \( m \) and radius \( r \) about an axis through its center is \( I = mr^2 \). The net torque that acts on the wheel due to the brake pads is, therefore,

\[
\Sigma \tau = I \alpha = (mr^2) \alpha
\]  

From Equation 9.1, the net torque that acts on the wheel due to the action of the two brake pads is

\[
\Sigma \tau = -2f_k \ell
\]  

where \( f_k \) is the kinetic frictional force applied to the wheel by each brake pad, and \( \ell = 0.33 \) m is the lever arm between the axle of the wheel and the brake pad (see the drawing in the text). The factor of 2 accounts for the fact that there are two brake pads. The minus sign arises because the net torque must have the same sign as the angular acceleration. The kinetic frictional force can be written as (see Equation 4.8)

\[
f_k = \mu_k F_N
\]
where $\mu_k$ is the coefficient of kinetic friction and $F_N$ is the magnitude of the normal force applied to the wheel by each brake pad.

Combining Equations (1), (2), and (3) gives

$$-2(\mu_k F_N) \ell = (mr^2)\alpha$$

$$F_N = \frac{mr^2 \alpha}{2\mu_k \ell} = \frac{-(1.3 \text{ kg})(0.33 \text{ m})^2 (-3.1 \text{ rad/s}^2)}{2(0.85)(0.33 \text{ m})} = 0.78 \text{ N}$$

40. **REASONING AND SOLUTION** The moment of inertia of a solid cylinder about an axis coinciding with the cylinder axis which contains the center of mass is $I_{cm} = (1/2)MR^2$. The parallel axis theorem applied to an axis on the surface (h = R) gives $I = I_{cm} + MR^2$, so that

$$I = \frac{3}{2}MR^2$$

41. **REASONING AND SOLUTION** For door A, $\theta = (1/2)\alpha_A t_A^2$. For door B, $\theta = (1/2)\alpha_B t_B^2$. Equating gives

$$t_B = t_A \sqrt{\frac{\alpha_A}{\alpha_B}}$$

We need the angular accelerations. Applying Newton’s second law for rotation to door A

$$FL = (1/3)ML^2\alpha_A$$

and to door B

$$F(1/2)L = (1/12)ML^2\alpha_B$$

Division of the previous two equations yields $\alpha_A/\alpha_B = 1/2$. Now

$$t_B = t_A / \sqrt{2} = (3.00 \text{ s})/(1.41) = 2.12 \text{ s}$$

42. **REASONING** The drawing shows the drum, pulley, and the crate, as well as the tensions in the cord
Let $T_1$ represent the magnitude of the tension in the cord between the drum and the pulley. Then, the net torque exerted on the drum must be, according to Equation 9.7, $\Sigma \tau = I_1 \alpha_1$, where $I_1$ is the moment of inertia of the drum, and $\alpha_1$ is its angular acceleration. If we assume that the cable does not slip, then Equation 9.7 can be written as

$$\frac{-T_1 r_1 + \tau}{\Sigma \tau} = \frac{G_{i1} r_1^2}{I_1} \frac{E_l}{I_k}$$

(1)

where $\tau$ is the counterclockwise torque provided by the motor, and $a$ is the acceleration of the cord ($a = 1.2 \text{ m/s}^2$). This equation cannot be solved for $\tau$ directly, because the tension $T_1$ is not known.

We next apply Newton’s second law for rotational motion to the pulley in the drawing:

$$\frac{+T_1 r_2 - T_2 r_2}{\Sigma \tau} = \frac{G_{m2} r_2^2}{I_2} \frac{E_l}{I_k}$$

(2)

where $T_2$ is the magnitude of the tension in the cord between the pulley and the crate, and $I_2$ is the moment of inertia of the pulley.

Finally, Newton’s second law for translational motion ($\Sigma F_y = m a$) is applied to the crate, yielding

$$\frac{+T_2 - m_3 g}{\Sigma F_y} = m_3 a$$

(3)
**SOLUTION** Solving Equation (1) for $T_1$ and substituting the result into Equation (2), then solving Equation (2) for $T_2$ and substituting the result into Equation (3), results in the following value for the torque

$$
\tau = r_1 \left[ a \Omega_1 + \frac{1}{2} m_2 + m_3 \mathbf{h} \cdot m_5 g \right] 
$$

$$
= (0.76 \text{ m}) \left[ (1.2 \text{ m} / \text{s}^2) \mathbf{c} 50 \text{ kg} + \frac{1}{2} 130 \text{ kg} + 180 \text{ kg} \mathbf{h} \cdot (180 \text{ kg})(9.80 \text{ m} / \text{s}^2) \right] = 1700 \text{ N} \cdot \text{m}
$$

---

43. **REASONING** The kinetic energy of the flywheel is given by Equation 9.9. The moment of inertia of the flywheel is the same as that of a solid disk, and, according to Table 9.1 in the text, is given by $I = \frac{1}{2} MR^2$. Once the moment of inertia of the flywheel is known, Equation 9.9 can be solved for the angular speed $\omega$ in rad/s. This quantity can then be converted to rev/min.

**SOLUTION** Solving Equation 9.9 for $\omega$, we obtain,

$$
\omega = \sqrt{\frac{2(KE_R)}{I}} = \sqrt{\frac{2(KE_R)}{\frac{1}{2} MR^2}} = \sqrt{\frac{4(1.2 \times 10^9 \text{ J})}{(13 \text{ kg})(0.30 \text{ m})^2}} = 6.4 \times 10^4 \text{ rad / s}
$$

Converting this answer into rev/min, we find that

$$
\omega = \frac{6.4 \times 10^4 \text{ rad / s}}{2 \pi \text{ rad / rev}} \times \frac{60 \text{ s}}{1 \text{ min}} = 6.1 \times 10^5 \text{ rev / min}
$$

---

44. **REASONING** The rotational kinetic energy of each object is given by Equation 9.9, $KE_R = \frac{1}{2} I \omega^2$. To solve this problem, we need only set the two kinetic energies equal, being sure to use the proper moment of inertia for each object, as given in Table 9.1.

**SOLUTION** According to Table 9.1 the moment of inertia of the solid sphere is $I_{\text{solid}} = \frac{2}{5} MR^2$, while the moment of inertia of the shell is $I_{\text{shell}} = \frac{2}{3} MR^2$. Using these expressions in Equation 9.9 for the rotational kinetic energy, we obtain
45. **REASONING AND SOLUTION**

a. The tangential speed of each object is given by Equation 8.9, \( v_T = r\omega \). Therefore,

For object 1: \( v_{T1} = (2.00 \text{ m})(6.00 \text{ rad/s}) = 12.0 \text{ m/s} \)

For object 2: \( v_{T2} = (1.50 \text{ m})(6.00 \text{ rad/s}) = 9.00 \text{ m/s} \)

For object 3: \( v_{T3} = (3.00 \text{ m})(6.00 \text{ rad/s}) = 18.0 \text{ m/s} \)

b. The total kinetic energy of this system can be calculated by computing the sum of the kinetic energies of each object in the system. Therefore,

\[
\text{KE} = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_3 v_3^2
\]

\[
\text{KE} = \frac{1}{2} [(6.00 \text{ kg})(12.0 \text{ m/s})^2 + (4.00 \text{ kg})(9.00 \text{ m/s})^2 + (3.00 \text{ kg})(18.0 \text{ m/s})^2] = 1.08 \times 10^3 \text{ J}
\]

c. The total moment of inertia of this system can be calculated by computing the sum of the moments of inertia of each object in the system. Therefore,

\[
I = \sum mr^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2
\]

\[
I = (6.00 \text{ kg})(2.00 \text{ m})^2 + (4.00 \text{ kg})(1.50 \text{ m})^2 + (3.00 \text{ kg})(3.00 \text{ m})^2 = 60.0 \text{ kg} \cdot \text{m}^2
\]

d. The rotational kinetic energy of the system is, according to Equation 9.9,

\[
\text{KE}_R = \frac{1}{2} I\omega^2 = \frac{1}{2} (60.0 \text{ kg} \cdot \text{m}^2)(6.00 \text{ rad/s})^2 = 1.08 \times 10^3 \text{ J}
\]

This agrees, as it should, with the result for part (b).
46. **REASONING**

a. The kinetic energy is given by Equation 9.9 as \( KE_R = \frac{1}{2} I \omega^2 \). Assuming the earth to be a uniform solid sphere, we find from Table 9.1 that the moment of inertia is \( I = \frac{2}{5} MR^2 \). The mass and radius of the earth is \( M = 5.98 \times 10^{24} \) kg and \( R = 6.38 \times 10^6 \) m (see the inside of the text’s front cover). The angular speed \( \omega \) must be expressed in rad/s, and we note that the earth turns once around its axis each day, which corresponds to \( 2\pi \) rad/day.

b. The kinetic energy for the earth’s motion around the sun can be obtained from Equation 9.9 as \( KE_R = \frac{1}{2} I \omega^2 \). Since the earth’s radius is small compared to the radius of the earth’s orbit (\( R_{\text{orbit}} = 1.50 \times 10^{11} \) m, see the inside of the text’s front cover), the moment of inertia in this case is just \( I = MR_{\text{orbit}}^2 \). The angular speed \( \omega \) of the earth as it goes around the sun can be obtained from the fact that it makes one revolution each year, which corresponds to \( 2\pi \) rad/year.

**SOLUTION**

a. According to Equation 9.9, we have

\[
KE_R = \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{2}{5} MR^2 \omega^2
\]

\[
= \frac{1}{2} \left[ \frac{2}{5} \times 5.98 \times 10^{24} \text{ kg} \times 6.38 \times 10^6 \text{ m} \right] \frac{2\pi \text{ rad}}{1 \text{ day}} \frac{1 \text{ day}}{24 \text{ h}} \frac{1 \text{ h}}{3600 \text{ s}}
\]

\[
= 2.57 \times 10^{29} \text{ J}
\]

b. According to Equation 9.9, we have

\[
KE_R = \frac{1}{2} I \omega^2 = \frac{1}{2} \frac{2}{5} MR_{\text{orbit}}^2 \omega^2
\]

\[
= \frac{1}{2} \frac{2}{5} \times 5.98 \times 10^{24} \text{ kg} \times \frac{1.50 \times 10^{11} \text{ m}}{2} \frac{2\pi \text{ rad}}{1 \text{ yr}} \frac{1 \text{ yr}}{365 \text{ day}} \frac{1 \text{ day}}{24 \text{ h}} \frac{1 \text{ h}}{3600 \text{ s}}
\]

\[
= 2.67 \times 10^{33} \text{ J}
\]

47. **REASONING AND SOLUTION** The conservation of energy gives for the cube \( (1/2) mv_c^2 = mgh \), so \( v_c = \sqrt{2gh} \). The conservation of energy gives for the marble \( (1/2) mv_m^2 + (1/2) I\omega^2 = mgh \). Since the marble rolls without slipping: \( \omega = v/R \). For a solid sphere we know \( I = (2/5) mR^2 \). Combining the last three equations gives
Thus,
\[ \frac{v_c}{v_m} = \sqrt{\frac{7}{5}} = 1.18 \]

---

48. **REASONING AND SOLUTION**  The rotational kinetic energy of the shell is \( KE_r = (1/2) I \omega^2 \) and the translational kinetic energy is \( KE_t = (1/2) M v^2 \). Now, \( v = R \omega \) since the sphere rolls without slipping, so
\[ KE_t = (1/2) MR^2 \omega^2 \]
The desired fraction is
\[ \frac{KE_r}{KE} = \frac{I}{I + MR^2} = \frac{2}{5} \]

---

49. **REASONING AND SOLUTION**  The only force that does work on the cylinders as they move up the incline is the conservative force of gravity; hence, the total mechanical energy is conserved as the cylinders ascend the incline. We will let \( h = 0 \) on the horizontal plane at the bottom of the incline. Applying the principle of conservation of mechanical energy to the solid cylinder, we have
\[ mgh_s = \frac{1}{2} mv_0^2 + \frac{1}{2} I_s \omega_0^2 \]  
(1)
where, from Table 9.1, \( I_s = \frac{1}{2} mr^2 \). In this expression, \( v_0 \) and \( \omega_0 \) are the initial translational and rotational speeds, and \( h_s \) is the final height attained by the solid cylinder. Since the cylinder rolls without slipping, the rotational speed \( \omega_0 \) and the translational speed \( v_0 \) are related according to Equation 8.12, \( \omega_0 = v_0 / r \). Then, solving Equation (1) for \( h_s \), we obtain
\[ h_s = \frac{3 v_0^2}{4g} \]
Repeating the above for the hollow cylinder and using \( I_h = mr^2 \) we have
\[ h_h = \frac{v_0^2}{g} \]
The height \( h \) attained by each cylinder is related to the distance \( s \) traveled along the incline and the angle \( \Theta \) of the incline by
\[
\begin{align*}
\text{s}_s &= \frac{h_s}{\sin \Theta} \quad \text{and} \quad \text{s}_h &= \frac{h_h}{\sin \Theta} \\
\text{Dividing these gives} &
\frac{s_s}{s_h} = \frac{3}{4}
\end{align*}
\]

50. **REASONING AND SOLUTION**  The conservation of energy gives

\[
mgh + \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} m v_0^2 + \frac{1}{2} I\omega_0^2
\]

If the ball rolls without slipping, \( \omega = \frac{v}{R} \) and \( \omega_0 = \frac{v_0}{R} \). We also know \( I = \frac{2}{5} mR^2 \).

Substitution of the last two equations into the first and rearrangement gives

\[
v = \sqrt{\frac{v_0^2}{2} - \frac{10}{7} gh} = \sqrt{0.50 \text{ m/s g} - \frac{10}{7} \circ 0.80 \text{ m/s }^2 \rate{m}} = 1.3 \text{ m/s}
\]

51. **REASONING**  We first find the speed \( v_0 \) of the ball when it becomes airborne using the conservation of mechanical energy. Once \( v_0 \) is known, we can use the equations of kinematics to find its range \( x \).

**SOLUTION**  When the tennis ball starts from rest, its total mechanical energy is in the form of gravitational potential energy. The gravitational potential energy is equal to \( mgh \) if we take \( h = 0 \) at the height where the ball becomes airborne. Just before the ball becomes airborne, its mechanical energy is in the form of rotational kinetic energy and translational kinetic energy. At this instant its total energy is \( \frac{1}{2} mv_0^2 + \frac{1}{2} I\omega^2 \). If we treat the tennis ball as a thin-walled spherical shell of mass \( m \) and radius \( r \), and take into account that the ball rolls down the hill without slipping, its total kinetic energy can be written as

\[
\frac{1}{2} mv_0^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} mv_0^2 + \frac{1}{2} \left( \frac{2}{3} mr^2 \right) \left( \frac{v_0}{r} \right)^2 = \frac{5}{6} mv_0^2
\]

Therefore, from conservation of mechanical energy, he have

\[
mgh = \frac{5}{6} mv_0^2 \quad \text{or} \quad v_0 = \sqrt{\frac{6gh}{5}}
\]

The range of the tennis ball is given by \( x = v_0 \sin \Theta \cdot t = v_0 (\cos \Theta) \cdot t \), where \( t \) is the flight time of the ball. From Equation 3.3b, we find that the flight time \( t \) is given by
Therefore, the range of the tennis ball is

\[ x = v_0 \cdot t = v_0 (\cos \Theta) \left( -\frac{2v_0 \sin \Theta}{a_y} \right) \]

If we take upward as the positive direction, then using the fact that \( a_y = -g \) and the expression for \( v_0 \) given above, we find

\[ x = \left( \frac{2 \cos \Theta \sin \Theta}{g} \right) v_0^2 = \left( \frac{2 \cos \Theta \sin \Theta}{g} \right) \left( \frac{6gh}{5} \right)^2 = \frac{12}{5} \cdot h \cos \Theta \cdot \sin \Theta \]

\[ = \frac{12}{5} (1.8 \, \text{m}) (\cos 35^\circ) (\sin 35^\circ) = 2.0 \, \text{m} \]

52. **REASONING AND SOLUTION** The angular momentum of the satellite is conserved if only the gravitational force of the earth is acting on it. We have \( L_A = L_P \) or \( mv_A r_A = mv_P r_P \). Now

\[ r_P = \left( \frac{v_A}{v_P} \right) r_A = \left( \frac{1}{1.20} \right) (1.30 \times 10^7 \, \text{m}) = 1.08 \times 10^7 \, \text{m} \]

53. **REASONING** Let the two disks constitute the system. Since there are no external torques acting on the system, the principle of conservation of angular momentum applies. Therefore we have \( L_{\text{initial}} = L_{\text{final}} \), or

\[ I_A \omega_A + I_B \omega_B = (I_A + I_B) \omega_{\text{final}} \]

This expression can be solved for the moment of inertia of disk B.

**SOLUTION** Solving the above expression for \( I_B \), we obtain

\[ I_B = I_A \left[ \frac{\omega_{\text{final}} - \omega_A}{\omega_B - \omega_{\text{final}}} \right] = (3.4 \, \text{kg} \cdot \text{m}^2) \left( \frac{-2.4 \, \text{rad/s} - 7.2 \, \text{rad/s}}{0.8 \, \text{rad/s} - (-2.4 \, \text{rad/s})} \right) = 4.4 \, \text{kg} \cdot \text{m}^2 \]

54. **REASONING AND SOLUTION** Angular momentum is conserved, \( I \omega = I_0 \omega_0 \) so
\[ \omega = \frac{\omega_0}{I_0} = \frac{40 \text{ kg} \cdot \text{m}^2}{180 \text{ kg} \cdot \text{m}^2} \times 7.11 \text{ rad/s} = 0.71 \text{ rad/s} \]

---

55. **REASONING AND SOLUTION**

a. Angular momentum is conserved, so that \( I\omega = I_0\omega_0 \), where

\[ I = I_0 + 10mb^2 = 2100 \text{ kg} \cdot \text{m}^2 \]

\[ \omega = \frac{(I_0/I)\omega_0}{0.20 \text{ rad/s}} = 0.14 \text{ rad/s} \]

b. A net external torque must be applied in a direction that is opposite to the angular deceleration caused by the baggage dropping onto the carousel.

---

56. **REASONING AND SOLUTION**

The conservation of angular momentum applies about the indicated axis

\[ I_d\omega_d + M_pR_p^2\omega_p = 0 \quad \text{where} \quad \omega_p = \frac{v_p}{R_p} \]

Now \( I_d = \frac{1}{2} M_dR_d^2 \). Combining and solving yields

\[ \omega_d = \frac{-2(M_p/M_d)(R_p/R_d^2)v_p}{0.500 \text{ rad/s}} \]

The magnitude of \( \omega_d \) is \( 0.500 \text{ rad/s} \). The negative sign indicates that the disk rotates in a direction opposite to the motion of the person.

---

57. **REASONING** Let the space station and the people within it constitute the system. Then as the people move radially from the outer surface of the cylinder toward the axis, any torques that occur are internal torques. Since there are no external torques acting on the system, the principle of conservation of angular momentum can be employed.

**SOLUTION** Since angular momentum is conserved,

\[ I_{\text{final}}\omega_{\text{final}} = I_0\omega_0 \]
Before the people move from the outer rim, the moment of inertia is

\[ I_0 = I_{\text{station}} + 500m_{\text{person}}r^2 \]

or

\[ I_0 = 3.00 \times 10^9 \text{ kg} \cdot \text{m}^2 + (500)(70.0 \text{ kg})(82.5 \text{ m})^2 = 3.24 \times 10^9 \text{ kg} \cdot \text{m}^2 \]

If the people all move to the center of the space station, the total moment of inertia is

\[ I_{\text{final}} = I_{\text{station}} = 3.00 \times 10^9 \text{ kg} \cdot \text{m}^2 \]

Therefore,

\[ \frac{\omega_{\text{final}}}{\omega_0} = \frac{I_0}{I_{\text{final}}} = \frac{3.24 \times 10^9 \text{ kg} \cdot \text{m}^2}{3.00 \times 10^9 \text{ kg} \cdot \text{m}^2} = 1.08 \]

This fraction represents a percentage increase of 8 percent.

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58. **REASONING** When the modules pull together, they do so by means of forces that are internal. These pulling forces, therefore, do not create a net external torque, and the angular momentum of the system is conserved. In other words, it remains constant. We will use the conservation of angular momentum to obtain a relationship between the initial and final angular speeds. Then, we will use Equation 8.9 \((v = r\omega)\) to relate the angular speeds \(\omega_0\) and \(\omega_f\) to the tangential speeds \(v_0\) and \(v_f\).

**SOLUTION** Let \(L\) be the initial length of the cable between the modules and \(\omega_0\) be the initial angular speed. Relative to the center-of-mass axis, the initial momentum of inertia of the two-module system is \(I_0 = 2M(L/2)^2\), according to Equation 9.6. After the modules pull together, the length of the cable is \(L/2\), the final angular speed is \(\omega_f\), and the momentum of inertia is \(I_f = 2M(L/4)^2\). The conservation of angular momentum indicates that

\[ \frac{I_f\omega_f}{I_0\omega_0} = \frac{I_f}{I_0} = \frac{2M(L/4)^2}{2M(L/2)^2} = \frac{1}{4} \]

According to Equation 8.9, \(\omega_f = v_f/(L/4)\) and \(\omega_0 = v_0/(L/2)\). With these substitutions, the result that \(\omega_f = 4\omega_0\) becomes
59. **REASONING AND SOLUTION** After the mass has moved inward to its final path the centripetal force acting on it is \( T = 105 \text{ N} \).

Its centripetal acceleration is \( a_c = \frac{v^2}{R} = \frac{T}{m} \)

\[
\frac{v_f}{L/4} = 4 \frac{v_0}{L/2} \text{ or } v_f = 2v_0 = 2 \text{ m/s} \text{ g = 34 m/s}
\]

The centripetal force is parallel to the line of action (the string), so the force produces no torque on the object. Hence, angular momentum is conserved.

\[
I\omega = I_o \omega_b \quad \text{so that} \quad \omega = (I_o/I)\omega_b = (R_o^2/R^2)\omega_b
\]

Substituting and simplifying

\[
R^3 = (mR_o^4 \omega_b^2)/T, \quad \text{so that} \quad R = 0.573 \text{ m}
\]

60. **REASONING AND SOLUTION** The block will just start to move when the centripetal force on the block just exceeds \( f_s^\text{max} \). Thus, if \( r_f \) is the smallest distance from the axis at which the block stays at rest when the angular speed of the block is \( \omega_f \), then \( \mu_s F_N = m r_f \omega_f^2 \), or

\[
\mu_s mg = m r_f \omega_f^2
\]

\[
\mu_s g = r_f \omega_f^2 \quad (1)
\]
Since there are no external torques acting on the system, angular momentum will be conserved.

\[ I_0 \omega_0 = I_f \omega_f \]

where \( I_0 = mr_0^2 \), and \( I_f = mr_f^2 \). Making these substitutions yields

\[ r_0^2 \omega_0 = r_f^2 \omega_f \]  

(2)

Solving Equation (2) for \( \omega_f \) and substituting into Equation (1) yields:

\[ \mu_s g = r_f \omega_0^2 \frac{r_0^4}{r_f^4} \]

Solving for \( r_f \) gives

\[ r_f = \left( \frac{\omega_0^2 r_0^4}{\mu_s g} \right)^{1/3} = \left[ \frac{(2.2 \text{ rad/s})^2 (0.30 \text{ m})^4}{(0.75)(9.80 \text{ m/s}^2)} \right]^{1/3} = 0.17 \text{ m} \]

61. **REASONING** The maximum torque will occur when the force is applied perpendicular to the diagonal of the square as shown. The lever arm \( \ell \) is half the length of the diagonal. From the Pythagorean theorem, the lever arm is, therefore,

\[ \ell = \frac{1}{2} \sqrt{(0.40 \text{ m})^2 + (0.40 \text{ m})^2} = 0.28 \text{ m} \]

Since the lever arm is now known, we can use Equation 9.1 to obtain the desired result directly.

**SOLUTION** Equation 9.1 gives

\[ \tau = F\ell = (15 \text{ N})(0.28 \text{ m}) = 4.2 \text{ N}\cdot\text{m} \]

62. **REASONING AND SOLUTION** For a solid disk we know \( I = (1/2)MR^2 \) so \( M = 2I/R^2 \).

Newton's second law for rotation yields \( I = (\Sigma \tau)/\alpha \), so \( M = 2(\Sigma \tau)/(\alpha R^2) \).

\[ \alpha = (\omega - \omega_0)/t = (-3.49 \text{ rad/s})/(15.0 \text{ s}) = -0.233 \text{ rad/s}^2 \]  

so that
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\[ M = \frac{2 \left(-6.20 \times 10^{-3} \text{ N} \cdot \text{m}\right)}{(-0.233 \text{ rad/s}^2)(0.150 \text{ m})^2} = 2.37 \text{ kg} \]

63. **REASONING AND SOLUTION** The sum of the forces must be zero:

\[ \Sigma F_y = F_W + F_C - mg = 0 \]

Therefore,

\[ F_W + F_C = mg \]

\[ = (71.0 \text{ kg})(9.80 \text{ m/s}^2) = 696 \text{ N} \]

The sum of the torques must also be zero. Taking the axis for torques at the point just below the boulder, we have

\[ \Sigma \tau = F_C (0.60 \text{ m}) - F_W (1.40 \text{ m}) = 0. \]

a. Substituting for \( F_W \) yields

\[ F_C (0.60 \text{ m}) - (696 - F_C)(1.40 \text{ m}) = 0 \]

Solving for \( F_C \) gives

\[ F_C = \boxed{487 \text{ N}} \]

b. Also, \( F_W = 696 \text{ N} - F_C = \boxed{209 \text{ N}} \)

64. **REASONING AND SOLUTION** Use conservation of angular momentum, \( I_o \omega_o = (I_o + I) \omega \), where

\[ I_o = 1.2 \times 10^{-3} \text{ kg} \cdot \text{m}^2 \quad \text{and} \quad \omega_o = 0.40 \text{ rad/s} \]

The moment of inertia of the bug is

\[ I = mL^2 = (5.0 \times 10^{-3} \text{ kg})(0.20 \text{ m})^2 = 2.0 \times 10^{-4} \text{ kg} \cdot \text{m}^2 \]

The final angular velocity is

\[ \omega = \frac{I_o \omega_o}{(I_o + I)} = \boxed{0.34 \text{ rad/s}} \]
65. **REASONING AND SOLUTION**
a. The rim of the bicycle wheel can be treated as a hoop. Using the expression given in Table 9.1 in the text, we have

\[ I_{\text{hoop}} = MR^2 = (1.20 \text{ kg})(0.330 \text{ m})^2 = 0.131 \text{ kg} \cdot \text{m}^2 \]

b. Any one of the spokes may be treated as a long, thin rod that can rotate about one end. The expression in Table 9.1 gives

\[ I_{\text{rod}} = \frac{1}{3} ML^2 = \frac{1}{3}(0.010 \text{ kg})(0.330 \text{ m})^2 = 3.6 \times 10^{-4} \text{ kg} \cdot \text{m}^2 \]

c. The total moment of inertia of the bicycle wheel is the sum of the moments of inertia of each constituent part. Therefore, we have

\[ I = I_{\text{hoop}} + 50I_{\text{rod}} = 0.131 \text{ kg} \cdot \text{m}^2 + 50(3.6 \times 10^{-4} \text{ kg} \cdot \text{m}^2) = 0.149 \text{ kg} \cdot \text{m}^2 \]

66. **REASONING AND SOLUTION** The net torque about an axis through the elbow joint is

\[ \Sigma \tau = (111 \text{ N})(0.300 \text{ m}) - (22.0 \text{ N})(0.150 \text{ m}) - (0.0250 \text{ m})M = 0 \quad \text{or} \quad M = 1.20 \times 10^{-3} \text{ N} \]

67. **REASONING** The drawing shows the forces acting on the board, which has a length \( L \). The ground exerts the vertical normal force \( V \) on the lower end of the board. The maximum force of static friction has a magnitude of \( \mu_s V \) and acts horizontally on the lower end of the board. The weight \( W \) acts downward at the board's center. The vertical wall applies a force \( P \) to the upper end of the board, this force being perpendicular to the wall since the wall is smooth (i.e., there is no friction along the wall). We take upward and to the right as our positive directions. Then, since the horizontal forces balance to zero, we have

\[ \mu_s V - P = 0 \quad \text{(1)} \]

The vertical forces also balance to zero giving

\[ V - W = 0 \quad \text{(2)} \]
Using an axis through the lower end of the board, we express the fact that the torques balance to zero as

\[ PL \sin \Theta - W - F_{HG} \cos \Theta = 0 \]  \hfill (3)

Equations (1), (2), and (3) may then be combined to yield an expression for \( \Theta \).

**SOLUTION** Rearranging Equation (3) gives

\[ \tan \Theta = \frac{W}{2P} \]  \hfill (4)

But, \( P = \mu_s V \) according to Equation (1), and \( W = V \) according to Equation (2). Substituting these results into Equation (4) gives

\[ \tan \Theta = \frac{V}{2\mu_s V} = \frac{1}{2\mu_s} \]

Therefore,

\[ \Theta = \tan^{-1} \left( \frac{1}{2\mu_s} \right) = \tan^{-1} \left( \frac{1}{2 \times 0.650} \right) = 37.6^\circ \]

68. **REASONING AND SOLUTION**
   a. The net torque about an axis through the point of contact between the floor and her shoes is

   \[ \Sigma \tau = - (5.00 \times 10^2 \text{ N})(1.10 \text{ m})\sin 30.0^\circ + F_N(\cos 30.0^\circ)(1.50 \text{ m}) = 0 \]

   \[ F_N = 212 \text{ N} \]

   b. Newton's second law applied in the horizontal direction gives \( F_h - F_N = 0 \), so \[ F_h = 212 \text{ N} \]

   c. Newton's second law in the vertical direction gives \( F_v - W = 0 \), so \[ F_v = 5.00 \times 10^2 \text{ N} \]

69. **REASONING AND SOLUTION** Since the change occurs without the aid of external torques, the angular momentum of the system is conserved: \( I_f \omega_f = I_0 \omega_0 \). Solving for \( \omega_f \) gives

\[ \omega_f = \omega_0 \left( \frac{I_0}{I_f} \right) \]
where for a rod of mass \( M \) and length \( L \), 
\[
I_0 = \frac{1}{12} ML^2 .
\]
To determine \( I_f \), we will treat the arms of the "u" as point masses with mass \( M/4 \) a distance \( L/4 \) from the rotation axis. Thus,
\[
I_f = \left[ \frac{1}{12} \left( \frac{M}{2} \right) \left( \frac{L}{2} \right)^2 \right] + \left[ 2 \left( \frac{M}{4} \right) \left( \frac{L}{4} \right)^2 \right] = \frac{1}{24} ML^2
\]
and
\[
\omega_f = \omega_0 \left[ \frac{1}{12} ML^2 \right] = (7.0 \text{ rad/s})(2) = 14 \text{ rad/s}
\]

70. **REASONING AND SOLUTION** The conservation of energy gives \( \frac{1}{2} I \omega^2 = mg[(1/2) L] \).

Now the tip of the rod has, as it hits the ground, a speed of \( v = L \omega \), so \( \omega = v/L \). The moment of inertia of the rod is given by \( I = (1/3) mL^2 \). Substituting the last two equations into the first equation and simplifying yields
\[
v = \sqrt{3gL} = \sqrt{3 \times 9.8 \text{ m/s}^2 \times 2 \text{ m}} = 7.67 \text{ m/s}
\]

71. **REASONING AND SOLUTION** Consider the left board, which has a length \( L \) and a weight of \( (356 \text{ N})/2 = 178 \text{ N} \). Let \( F_v \) be the upward normal force exerted by the ground on the board. This force balances the weight, so \( F_v = 178 \text{ N} \). Let \( f_s \) be the force of static friction, which acts horizontally on the end of the board in contact with the ground. \( f_s \) points to the right. Since the board is in equilibrium, the net torque acting on the board through any axis must be zero. Measuring the torques with respect to an axis through the apex of the triangle formed by the boards, we have
\[
+ (178 \text{ N})(\sin 30.0^\circ) \left( \frac{L}{2} \right) + f_s(L \cos 30.0^\circ) - F_v(L \sin 30.0^\circ) = 0
\]
or
\[
44.5 \text{ N} + f_s \cos 30.0^\circ - F_v \sin 30.0^\circ = 0
\]
so that
\[
f_s = \frac{(178 \text{ N})(\sin 30.0^\circ) - 44.5 \text{ N}}{\cos 30.0^\circ} = 51.4 \text{ N}
\]

72. **REASONING AND SOLUTION** Newton's law applied to the 11.0-kg object gives
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\[ T_2 - (11.0 \text{ kg})(9.80 \text{ m/s}^2) = (11.0 \text{ kg})(4.90 \text{ m/s}^2) \quad \text{or} \quad T_2 = 162 \text{ N} \]

A similar treatment for the 44.0-kg object yields

\[ T_1 - (44.0 \text{ kg})(9.80 \text{ m/s}^2) = (44.0 \text{ kg})(-4.90 \text{ m/s}^2) \quad \text{or} \quad T_1 = 216 \text{ N} \]

For an axis about the center of the pulley

\[ T_2r - T_1r = I(-\alpha) = (1/2) Mr^2(-a/r) \]

Solving for the mass \( M \) we obtain

\[ M = (-2/a)(T_2 - T_1) = [-2/(4.90 \text{ m/s}^2)](162 \text{ N} - 216 \text{ N}) = 22.0 \text{ kg} \]

---

73. **CONCEPT QUESTIONS**

a. The magnitude of a torque is the magnitude of the force times the lever arm of the force, according to Equation 9.1. The lever arm is the perpendicular distance between the line of action of the force and the axis. For the force at corner B, the lever arm is half the length of the short side of the rectangle. For the force at corner D, the lever arm is half the length of the long side of the rectangle. Therefore, the lever arm for the force at corner D is greater. Since the magnitudes of the two forces are the same, this means that the force at corner D creates the greater torque.

b. As we have seen, the torque at corner D (clockwise) has a greater magnitude than the torque at corner B (counterclockwise). Therefore, the net torque from these two contributions is clockwise. This means that the force at corner A must produce a counterclockwise torque, if the net torque produced by the three forces is to be zero. The force at corner A, then, must point toward corner B.

**SOLUTION** Let \( L \) be the length of the short side of the rectangle, so that the length of the long side is \( 2L \). We can use Equation 9.1 for the magnitude of each torque. Remembering that counterclockwise torques are positive and that the torque at corner A is counterclockwise, we can write the zero net torque as follows:

\[ \Sigma \tau = F_A \ell_A + F_B \ell_B - F_D \ell_D = 0 \]

\[ F_A L + b g - h b g = 0 \]

The length \( L \) can be eliminated algebraically from this result, which can then be solved for \( F_A \):

\[ F_A = -b g + h g = 6.0 \text{ N (pointing toward corner B)} \]
74. **CONCEPT QUESTIONS**  
a. In the left design, both the 60.0 and the 525-N forces create clockwise torques with respect to the rotational axis at the point where the tire contacts the ground. In the right design, only the 60.0-N force creates a torque, because the line of action of the 525-N force passes through the axis. Thus, the total torque from the two forces is greater for the left design.

b. The man is supporting the wheelbarrow in equilibrium. Therefore, his force must create a torque that balances the total torque from the other two forces. To do this, his torque must be counterclockwise and have the same magnitude as the total torque from the other two forces. Since the two forces create more torque in the left design, the magnitude of the man’s torque must be greater for that design.

c. The magnitude of a torque is the magnitude of the force times the lever arm of the force, according to Equation 9.1. The drawing shows that the lever arms of the man’s forces in the two designs is the same (1.300 m). Thus, to create the greater torque necessitated by the left design, the man must apply a greater force for that design.

**SOLUTION**  
The lever arms for the forces can be obtained from the distances shown in the drawing for each design. We can use Equation 9.1 for the magnitude of each torque. Remembering that counterclockwise torques are positive, we can write an expression for the zero net torque for each design. These expressions can be solved for the man’s force $F$ in each case:

**Left design**

\[ \Sigma \tau = -525 \text{N} \cdot 0.400 \text{ m} + 60.0 \text{N} \cdot 0.600 \text{ m} - F \cdot 1.300 \text{ m} = 0 \]

\[ F = \frac{525 \text{N} \cdot 0.400 \text{ m} - 60.0 \text{N} \cdot 0.600 \text{ m}}{1.300 \text{ m}} = 189 \text{ N} \]

**Right design**

\[ \Sigma \tau = -60.0 \text{N} \cdot 0.600 \text{ m} - F \cdot 1.300 \text{ m} = 0 \]

\[ F = \frac{60.0 \text{N} \cdot 0.600 \text{ m}}{1.300 \text{ m}} = 27.7 \text{ N} \]

75. **CONCEPT QUESTIONS**  
a. The forces in part $b$ of the drawing cannot keep the beam in equilibrium (no acceleration), because neither $V$ nor $W$, both being vertical, can balance the horizontal component of $P$. This unbalanced component would accelerate the beam to the right.

b. The forces in part $c$ of the drawing cannot keep the beam in equilibrium (no acceleration). To see why, consider a rotational axis perpendicular to the plane of the paper and passing through the pin. Neither $P$ nor $H$ create torques relative to this axis, because their lines of action pass directly through it. However, $W$ does create a torque, which is unbalanced. This torque would cause the beam to have a counterclockwise angular acceleration.
c. The forces in part d are sufficient to eliminate the accelerations discussed above, so they can keep the beam in equilibrium.

**SOLUTION** Assuming that upward and to the right are the positive directions, we obtain the following expressions by setting the sum of the vertical and the sum of the horizontal forces equal to zero:

**Horizontal forces** \[ P \cos \theta - H = 0 \] (1)

**Vertical forces** \[ P \sin \theta + V - W = 0 \] (2)

Using a rotational axis perpendicular to the plane of the paper and passing through the pin and remembering that counterclockwise torques are positive, we also set the sum of the torques equal to zero. In doing so, we use L to denote the length of the beam and note that the lever arms for W and V are L/2 and L, respectively. The forces P and H create no torques relative to this axis, because their lines of action pass directly through it.

**Torques** \[ W \ell L - VH = 0 \] (3)

Since L can be eliminated algebraically, Equation (3) may be solved immediately for V:

\[ V = \frac{1}{2} W = \frac{1}{2} \times 40 \text{ N} = 20 \text{ N} \]

Substituting this result into Equation (2) gives

\[ P \sin \theta + \frac{1}{2} W - W = 0 \]

\[ P = \frac{W}{2 \sin \theta} = \frac{340 \text{ N}}{2 \sin 39^\circ} = 270 \text{ N} \]

Substituting this result into Equation (1) gives

\[ \frac{W}{2 \cos \theta} \cos \theta - H = 0 \]

\[ H = \frac{W}{2 \tan \theta} = \frac{340 \text{ N}}{2 \tan 39^\circ} = 210 \text{ N} \]

---

76. **CONCEPT QUESTIONS**

a. Equation 8.7 from the equations of rotational kinematics indicates that the angular displacement \( \Theta \) is \( \Theta = \omega_0 t + \frac{1}{2} \alpha t^2 \), where \( \omega_0 \) is the initial angular velocity, \( t \) is the
time, and $\alpha$ is the angular acceleration. Since both wheels start from rest, $\omega_0 = 0 \text{ rad/s}$ for each. Furthermore, each wheel makes the same number of revolutions in the same time, so $\Theta$ and $t$ are also the same for each. Therefore, the angular acceleration $\alpha$ must be the same for each.

b. For the hoop, all of the mass is located at a distance from the axis equal to the radius. For the disk, some of the mass is located closer to the axis. According to Equation 9.6, the moment of inertia is greater when the mass is located farther from the axis. Therefore, the moment of inertia of the hoop is greater. Table 9.1 indicates that the moment of inertia of a hoop is $I_{\text{hoop}} = MR^2$, while the moment of inertia of a disk is $I_{\text{disk}} = \frac{1}{2} MR^2$.

c. Newton’s second law for rotational motion indicates that the net external torque is equal to the moment of inertia times the angular acceleration. Both disks have the same angular acceleration, but the moment of inertia of the hoop is greater. Thus, the net external torque must be greater for the hoop.

**SOLUTION** Using Equation 8.7, we can express the angular acceleration as follows:

$$\Theta = \omega_0 t + \frac{1}{2} \alpha t^2 \quad \text{or} \quad \alpha = \frac{2(\Theta - \omega_0 t)}{t^2}$$

This expression for the acceleration can now be used in Newton’s second law for rotational motion. Accordingly, the net external torque is

**Hoop** \[ \Sigma \tau = I_{\text{hoop}} \alpha = MR^2 \left( \frac{2(\Theta - \omega_0 t)}{t^2} \right) \]

\[ = 0.0 \text{ kg} \cdot 0.35 \text{ m} \cdot 2.0 \text{ rad/s} \cdot 2.0 \text{ rad/s} \cdot 0.0 \text{ s} \cdot 0.20 \text{ N} \cdot \text{m} \]

**Disk** \[ \Sigma \tau = I_{\text{disk}} \alpha = \frac{1}{2} MR^2 \left( \frac{2(\Theta - \omega_0 t)}{t^2} \right) \]

\[ = \frac{1}{2} \cdot 0.0 \text{ kg} \cdot 0.35 \text{ m} \cdot 2.0 \text{ rad/s} \cdot 2.0 \text{ rad/s} \cdot 0.0 \text{ s} \cdot 0.10 \text{ N} \cdot \text{m} \]

77. **CONCEPT QUESTIONS** a. According to Equation 9.6, the moment of inertia for rod A is just that of the attached particle, since the rod itself is massless. For rod A with its attached particle, then, the moment of inertia is $I_A = ML^2$. According to Table 9.1, the moment of inertia for rod B is $I_B = \frac{1}{3} ML^2$. The moment of inertia for rod A with its attached particle is greater.
b. Since the moment of inertia for rod A is greater, its kinetic energy is also greater, according to Equation 9.9 $KE_R = \frac{1}{2} I \omega^2$.

**SOLUTION** Using Equation 9.9 to calculate the kinetic energy, we find

\[
Rod A: KE_R = \frac{1}{2} I_A \omega^2 = \frac{1}{2} M L^2 \omega^2 = \frac{1}{2} \cdot 6.66 \text{ kg} \cdot 0.75 \text{ m} \cdot 2 \text{ rad/s} = 3.3 \text{ J}
\]

\[
Rod B: KE_R = \frac{1}{2} I_B \omega^2 = \frac{1}{2} M L^2 \omega^2 = \frac{1}{6} \cdot 6.66 \text{ kg} \cdot 0.75 \text{ m} \cdot 2 \text{ rad/s} = 1.1 \text{ J}
\]

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78. **CONCEPT QUESTIONS**

a. The force is applied to the person in the counterclockwise direction by the bar that he grabs when climbing aboard.

b. According to Newton’s action-reaction law, the person must apply a force of equal magnitude and opposite direction to the bar and the carousel. This force acts on the carousel in the clockwise direction and, therefore, creates a clockwise torque.

c. Since the carousel is moving counterclockwise, the clockwise torque applied to it when the person climbs aboard decreases its angular speed.

**SOLUTION** We consider a system consisting of the person and the carousel. Since the carousel rotates on frictionless bearings, no net external torque acts on this system. Therefore, angular momentum is conserved, and we can set the total angular momentum of the system before the person hops on equal to the total angular momentum afterwards. Afterwards, the angular momentum includes a contribution from the person. According to Equation 9.6, his moment of inertia is $I_{\text{person}} = MR^2$, since he is at the outer edge of the carousel.

\[
I_{\text{carousel}} \omega_f + I_{\text{person}} \omega_f = I_{\text{carousel}} \omega_0
\]

\[
\omega_f = \frac{I_{\text{carousel}} \omega_0}{I_{\text{carousel}} + I_{\text{person}}} = \frac{I_{\text{carousel}} \omega_0}{I_{\text{carousel}} + MR^2}
\]

\[
= \frac{25 \text{ kg} \cdot \text{m}^2 \cdot \omega_f}{125 \text{ kg} \cdot \text{m}^2 + 0.0 \text{ kg} \cdot 0.50 \text{ m}^2} = 1.83 \text{ rad/s}
\]